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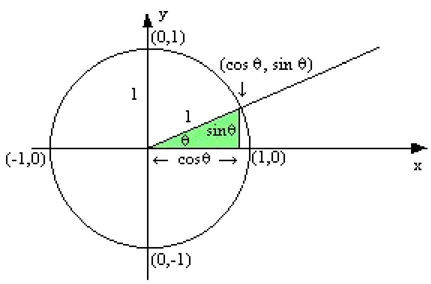
Circles are Triangles… Trust us

For our trig project we produced an interactive unit-circle out of plywood and yarn. We chose this project to illuminate the following statements:

1. Circles are triangles
2. (cos, sin) = (x, y)

For the first statement, circles can be broken down into any type of triangles; for ease of computation we'll use two main types of triangles: 30-60-90, and 45-45-90. The 30-60-90 triangles can be reflected and rotated around the axes 8 different times, while the 45-45-90 triangles can be reflected and rotated to 4 different places on the unit-circle. Because the radius of the unit circle is 1, every triangle found in the circle has a hypotenuse of one.

For the second statement (cos,sin)=(x,y). This means that in the coordinate plane, cos is equivalent to the x value, the same is true with sin and y.



In the figure on the left, one can see that the coordinate points (cos,sin) aren't just arbitrary points, but are derived using sin and cos and their relation to the hypotenuse. (Which is the same as the radius of the circle.)

The height of this triangle is equivalent to the opposite side over the hypotenuse (sin / 1).

Image taken from: http://www.afralisp.net/archive/images/l4.GIF

The formulae that are shown in our piece are shown below:

1. $Sin^2 + Cos^2 = 1$
    a. The radius of the unit circle is 1, so all of the triangles hypotenuses are 1. This is in accordance to the Pythagorean Theorem: $a^2 + b^2 = c^2$. If $c^2 = 1$, then a and b are the other two sides of the triangle; which is the same as sin / 1 and cos / 1.
    b. Sin describes the triangle's side on the Y axis and cos describes the side on the X axis, so X^2+Y^2= the hypotenuse (1), $cos^2+sin^2=1$.

2. sin(-x) = -sin(x), cos(-x) = cos(x)
    a. We are using yarn to model how triangles are reflected across the axis.
        i. When the triangles are reflected across the y-axis, or vertical axis (created by changing the sign of the angle), it's easy to see that sin(-x) = -sin(x). (The negative sign denotes the height of the triangle.)
        ii. When the triangles are reflected across the x-axis (horizontal axis), it's easy to see that cos(-x) = cos(x).

Directions:

1. Try making a 30-60-90 degree triangle at $\frac{\pi}{6}$ radians. Now make a 30-60-90 degree triangle at $\frac{11\pi}{6}$ radians. What do you notice about the two triangles' sine values?

   Answer: These two triangles now represent the equation: sin(-x) = -sin(x). (sin = y-value… so... sin of $\frac{\pi}{6} = \frac{1}{2}$ & sin of $\frac{11\pi}{6} = -\frac{1}{2}$)

2. Try making a 45-45-90 degree triangle at 135º. Now make a 45-45-90 degree triangle at 225º. What do you notice about the two triangles' cosine values?

   Answer: Because this is a reflection over the x-axis, these two triangles now represent the equation: cos(-x) = cos(x). (cos = x-value.... so… cos of 135º = $-\frac{\sqrt{2}}{2}$ & cos of 225º = $-\frac{\sqrt{2}}{2}$)

3. Try making a triangle at 60º. Now try making a triangle at 780º. Compare the two triangles, what do you notice?

   Answer: They're the same! This is modelled by the equation: sin(x) = sin(x+k2π)